Name ______________________________ Date ______________

Introduction to Matrices

A **matrix** is a rectangular array of numbers written within brackets. A matrix is identified by a capital letter. A matrix is classified by its dimensions—the number of columns and rows it contains.

Matrix X to the right has 3 rows and 2 columns. It is a 3×2 matrix.

A **matrix element** is a number in the matrix. Each matrix element is identified by its location within the matrix.

element X_{12}

$$X = \begin{bmatrix} 29{,}300 & 2{,}900 \\ 23{,}200 & 2{,}100 \\ 15{,}400 & 1{,}200 \end{bmatrix} \text{ 3 rows}$$

2 columns

Rules for Reading a Matrix

1. The dimensions of a matrix are given in terms of rows and columns.

2. A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by row and column.

Example

State the dimensions of the matrix. Identify element A_{23}. $A = \begin{bmatrix} 4 & 5 & 6 \\ -1 & 0 & 2 \end{bmatrix}$

Step 1 The dimensions of a matrix are given in terms of rows and columns.

The matrix has 2 rows and 3 columns; it is a 2×3 matrix.

Step 2 A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by the row and column.

A_{23} is the element in row 2, column 3. $A_{23} = 2$

Practice A

State the dimensions of the matrix. Identify the specified element.

1. Identify element B_{22}.

$B = \begin{bmatrix} 3 & 9 & 1 & 6 \\ 0 & 7 & 9 & 7 \end{bmatrix}$

The dimensions of a matrix are given in terms of rows and columns.

The matrix has ______ rows and ______ columns; it is a ________ matrix.

A matrix element is identified by (1) using the letter of the matrix, and (2) using a subscript to identify the position of the element by the row and column.

B_{22} is the element in row ______, column ______. B_{22} = ______

2. Identify element Z_{21}. $Z = \begin{bmatrix} 10 & 0 \\ -2 & 1 \end{bmatrix}$ ____________________

3. Identify the location of -10. $Z = \begin{bmatrix} 0 & -1 & -4 & 5 \\ 3 & 5 & -10 & 7 \\ 6 & -3 & -1 & 0 \end{bmatrix}$ ____________________

Name ______________________ Date ______________

Matrix Addition

When adding matrices, you add the corresponding elements in each matrix.

corresponding elements

$$\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -4 & 7 \end{bmatrix}$$

Rule for Matrix Addition
Add corresponding elements in each matrix to form one large matrix.

Example

Add. $\begin{bmatrix} \mathbf{-4} & \mathbf{2} \\ \mathbf{-10} & \mathbf{7} \end{bmatrix} + \begin{bmatrix} \mathbf{5} & \mathbf{-9} \\ \mathbf{9} & \mathbf{-3} \end{bmatrix}$

Add corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} 4 & 2 \\ -10 & 7 \end{bmatrix} + \begin{bmatrix} 5 & -9 \\ 9 & -3 \end{bmatrix} = \begin{bmatrix} 4+5 & 2+(-9) \\ (-10)+9 & 7+(-3) \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -7 \\ -1 & 4 \end{bmatrix}$$

Practice B

Add.

1. $\begin{bmatrix} \mathbf{-5} & \mathbf{8} \\ \mathbf{3} & \mathbf{-3} \end{bmatrix} \quad \begin{bmatrix} \mathbf{11} & \mathbf{-1} \\ \mathbf{5} & \mathbf{-7} \end{bmatrix}$

Add corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} -5 & 8 \\ 3 & -3 \end{bmatrix} + \begin{bmatrix} 11 & -1 \\ 5 & -7 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

2. $\begin{bmatrix} 2 & -9 & -4 \\ 3 & -1 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 16 & 9 \\ 11 & 1 & 2 \end{bmatrix}$ ______________________

3. $\begin{bmatrix} -4 & 7 \\ -9 & 3 \end{bmatrix} + \begin{bmatrix} 7 & 2 \\ -4 & 20 \end{bmatrix}$ ______________________

4. $\begin{bmatrix} 2 \\ 17 \end{bmatrix} + \begin{bmatrix} 5 \\ -12 \end{bmatrix}$ ______________________

Name ______________________ Date ______________

Matrix Subtraction

When subtracting matrices, you subtract the corresponding elements in each matrix.

corresponding elements

$$\begin{bmatrix} -2 & 0 \\ 4 & 2 \end{bmatrix} \quad \begin{bmatrix} 3 & -1 \\ -4 & 7 \end{bmatrix}$$

Rule for Matrix Subtraction
Subtract corresponding elements in each matrix to form one large matrix.

Example
Subtract. $\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 6 \\ 8 & 5 \end{bmatrix}$

Subtract corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} -2 & 5 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} -4 & 6 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} -2-(-4) & 5-6 \\ 0-8 & -2-5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -8 & -7 \end{bmatrix}$$

Practice C

Subtract.

1. $\begin{bmatrix} 3 & 3 \\ -4 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 8 & -2 \end{bmatrix}$

Subtract corresponding elements in each matrix to form one large matrix.

$$\begin{bmatrix} 3 & 3 \\ -4 & -1 \end{bmatrix} - \begin{bmatrix} 6 & -2 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

$$= \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

2. $\begin{bmatrix} -3 & 5 \\ 0 & -4 \end{bmatrix} - \begin{bmatrix} -5 & 9 \\ 10 & 3 \end{bmatrix}$ ______________________

3. $\begin{bmatrix} 9 & -12 & 15 \\ 4 & 7 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 4 & -3 \\ 5 & -1 & -3 \end{bmatrix}$ ______________________

4. $\begin{bmatrix} 6 & 12 \\ -8 & -4 \end{bmatrix} - \begin{bmatrix} -9 & 0 \\ -2 & -9 \end{bmatrix}$ ______________________

Name ______________________ Date ______________

Scalar Multiplication

A **matrix** is a rectangular arrangement of numbers in rows and columns. You can think of a matrix as a way to organize data, similar to the way data is displayed in a table. A **scalar** is a real number factor by which all the elements of a matrix are multiplied.

> **Rule for Scalar Multiplication**
> Create an expanded matrix by multiplying each element by the scalar.

Example

Solve. $2\begin{bmatrix} -6 & 4 \\ 7 & -3 \end{bmatrix}$

Create an expanded matrix by multiplying each element by the scalar.

$$2\begin{bmatrix} -6 & 4 \\ 7 & -3 \end{bmatrix} = \begin{bmatrix} -6 \times 2 & 4 \times 2 \\ 7 \times 2 & -3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12 & 8 \\ 14 & -6 \end{bmatrix}$$

Practice D

Solve.

1. $5\begin{bmatrix} 11 & -9 & -4 \\ -5 & 6 & 3 \end{bmatrix}$

Create an expanded matrix by multiplying each element by the scalar.

$$5\begin{bmatrix} 11 & -9 & -4 \\ -5 & 6 & 3 \end{bmatrix} = \begin{bmatrix} 11 \times 5 & \_\_\_ & \_\_\_ \\ -5 \times 5 & \_\_\_ & \_\_\_ \end{bmatrix}$$

$$= \begin{bmatrix} 55 & \_\_\_ & \_\_\_ \\ -25 & \_\_\_ & \_\_\_ \end{bmatrix}$$

2. $-3\begin{bmatrix} 2 & 16 \\ 9 & -2 \\ -11 & 6 \end{bmatrix}$ ______________________

3. $4\begin{bmatrix} 5 & -12 \\ 8 & -2 \end{bmatrix}$ ______________________

4. $-6\begin{bmatrix} -8 & -4 & 1 \\ 0 & 2 & -9 \end{bmatrix}$ ______________________

Name ______________________ Date ______________

Matrix Multiplication

When multiplying matrices, you multiply the elements of a row in the first matrix by the corresponding elements in a column of the second matrix. You then add the products.

$$\begin{bmatrix} 3 & 6 & 5 \\ 5 & 3 & 8 \end{bmatrix} \times \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \times 5 & + & 6 \times 2 & + & 5 \times 2 \\ 5 \times 5 & + & 3 \times 2 & + & 8 \times 2 \end{bmatrix}$$

Rules for Matrix Multiplication

1. Circle each row of the first matrix; circle each column of the second matrix.
2. Multiply the elements of a row in the first matrix by the elements of each column in the second matrix. Add the products in each row.

The dimensions of the resulting matrix will be the number of rows in the first matrix by the number of columns in the second matrix.

Example

Multiply. $\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

Step 1 Identify the elements to be multiplied.

$$\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

Step 2 Multiply the elements of a row in the first matrix by the elements of each column in the second matrix.

$$\begin{bmatrix} 3 & 5 \\ 2 & -3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 5 \times 9 \\ 2 \times 2 + -3 \times 9 \end{bmatrix}$$

Add the products in each row.

$$= \begin{bmatrix} 6 + 45 \\ 4 + -27 \end{bmatrix} = \begin{bmatrix} 51 \\ -23 \end{bmatrix}$$

Practice E

Multiply.

1. $\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix}$

Identify the elements to be multiplied.

$$\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix}$$

Multiply the elements of a row in the first matrix by the elements of each column in the second matrix.

$$\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} \times \begin{bmatrix} 1 & 9 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

Add the products in each row.

$$= \begin{bmatrix} \quad \end{bmatrix}$$

2. $\begin{bmatrix} 6 & 3 & 8 \\ 9 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 7 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$

3. $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 9 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$

Name ______________________ Date ______________________

Writing the Inverse of a Matrix

A **matrix** is a rectangular array of numbers written within brackets. A matrix is identified by a capital letter. You will use the inverse of a matrix to help solve systems of equations.

Rules for Finding the Inverse of a 2 × 2 Matrix

1. In a 2 × 2 matrix, assign a, b, c, and d to the elements in the matrix as follows: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$
2. Find the value of $ad - bc$.
3. Plug the values from Steps 1 and 2 into the formula $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. Simplify.

Example

Find the inverse of $\begin{bmatrix} 3 & -1 \\ 7 & 1 \end{bmatrix}$.

Step 1 In a 2 × 2 matrix, assign a, b, c, and d to the elements in the matrix. $a = 3, b = -1, c = 7, d = 1$

Step 2 Find the value of $ad - bc$. $ad - bc = (3)(1) - (-1)(7) = 3 + 7 = 10$

Step 3 Plug the values from Steps 1 and 2 into the formula. Simplify. $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 1 \\ -7 & 3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.1 \\ -0.7 & 0.3 \end{bmatrix}$

Practice F

Find the inverse of each matrix.

1. $\begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix}$

In a 2 × 2 matrix, assign a, b, c, and d to the elements in the matrix. $a =$ _____, $b =$ _____, $c =$ _____, $d =$ _____

Find the value of $ad - bc$. $ad - bc =$ _______ − _______ = _____

Plug the values from Steps 1 and 2 into the formula. Simplify. $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{}{\_\_\_\_}\begin{bmatrix} & \\ & \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$

2. $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} & \\ & \end{bmatrix}$

4. $\begin{bmatrix} 8 & 2 \\ 7 & 3 \end{bmatrix}$ $\begin{bmatrix} & \\ & \end{bmatrix}$

3. $\begin{bmatrix} -2 & -5 \\ -3 & -8 \end{bmatrix}$ $\begin{bmatrix} & \\ & \end{bmatrix}$

5. $\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$ $\begin{bmatrix} & \\ & \end{bmatrix}$

Name ______________________________ Date ______________

Solving a Matrix Equation

A **matrix** is a rectangular array of numbers written within brackets. A matrix is identified by a capital letter. You can use the inverse of a matrix to solve matrix equations.

Rules for Solving a Matrix Equation

1. A matrix equation has the form $AX = B$. Find the inverse of the matrix A.
2. Solve for the unknown, matrix X: Multiply matrix B by the inverse of matrix A. Then simplify.

Example

Solve. $\begin{bmatrix} -11 & 25 \\ 4 & -9 \end{bmatrix} X = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$

Step 1 Find the inverse of matrix A.

$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\frac{1}{(9 \times 11)-(25 \times 4)}\begin{bmatrix} -9 & -25 \\ -4 & -11 \end{bmatrix} = \begin{bmatrix} 9 & 25 \\ 4 & 11 \end{bmatrix}$$

Step 2 Solve for the unknown, matrix X: Multiply matrix B by the inverse of matrix A. Then simplify.

$$X = \begin{bmatrix} 9 & 25 \\ 4 & 11 \end{bmatrix}\begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

$$X = \begin{bmatrix} 9(3) + 25(-7) \\ 4(3) + 11(-7) \end{bmatrix} = \begin{bmatrix} -148 \\ -65 \end{bmatrix}$$

Practice G

1. $\begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix} X = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$

Find the inverse of matrix A.

$$\frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\frac{1}{\_\_\_\_ - \_\_\_\_}\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

Solve for the unknown, matrix X: Multiply matrix B by the inverse of matrix A. Then simplify.

$$X = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

$$X = \begin{bmatrix} \quad + \quad \\ \quad + \quad \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \end{bmatrix}$$

2. $\begin{bmatrix} -3 & -4 \\ 8 & 11 \end{bmatrix} X = \begin{bmatrix} -2 \\ 6 \end{bmatrix}$ ____________

3. $\begin{bmatrix} -5 & -3 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ ____________