

Unit 15 Lesson 1: Law of Sines

In this lesson you will:

- Understand the concept of the Law of Sines
- Apply the Law of Sines formula to calculate the values of angles in a triangle

This is the law of sines. For any triangle, the following is true.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Using this formula, you can find values for unknown angles and sides when given some of the values of the triangle.

To find unknown sides, use the formula above.

To find unknown angles, use the opposite formula below.

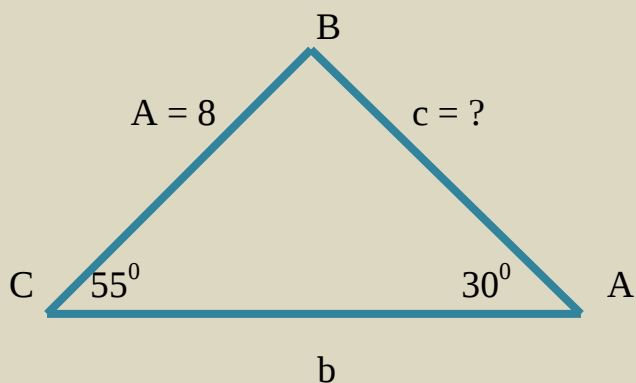
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You don't have to have all three sets of the values above. Use the ones that you have. Substitute given values, then solve for unknown values using algebra.

Use a scientific calculator for these problems. There are many online calculators that you can use, and the Windows calculator will work just fine. You can work through the problem on your calculator without rounding. If you stop between steps and round, your answer may vary slightly.

There are some rules you need to remember:

- Angles are named with upper case letters (example: $\angle M$)
- Sides are named with lower case letters (example: m)
- Sides are opposite their angle (example: side ' m ' is opposite $\angle M$)
- The sum of the three angle measures inside a triangle equals 180°
- To find the angle measure of one unknown angle, you must add the two known angle measures then subtract that value from 180°
- To find the sin value of an angle, enter the angle measure into the calculator, then hit 'sin'
- To find the value of an angle when given the sin value, enter the sin value into the calculator, then hit 'inv' then hit 'sin'



A. In $\triangle ABC$, side $a = 8$, $m\angle A = 30^\circ$ and $m\angle C = 55^\circ$. Find side c to the nearest tenth of an integer.

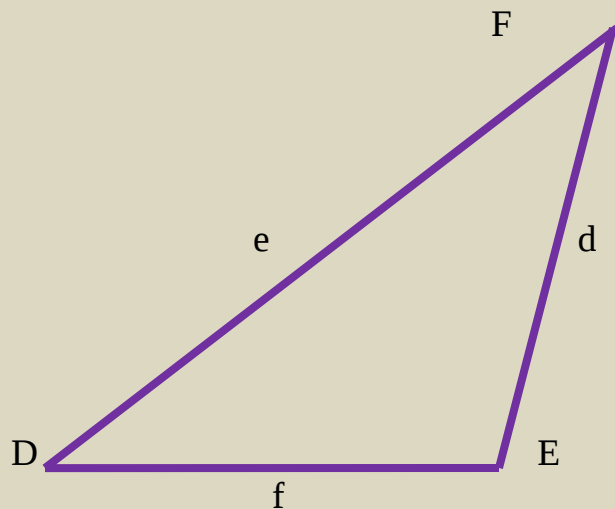
Use the Law of Sines, substituting the values that you are given. Here you are given side a , $\angle A$, and $\angle C$. So, use those values.

$$a/\sin A = c/\sin C \rightarrow 8/\sin 30^\circ = c/\sin 55^\circ$$

$$(\sin 30^\circ) * c = 8(\sin 55^\circ)$$

$$\frac{c = 8(\sin 55^\circ)}{\sin 30^\circ} \approx 13.1$$

Answer: 13.1



B. In $\triangle DEF$, $\sin F = 1/5$, $\sin D = 2/5$, and $f = 24$. Find the length of side d .

Again, we are working with two sides and two angles.

Use the Law of Sines:

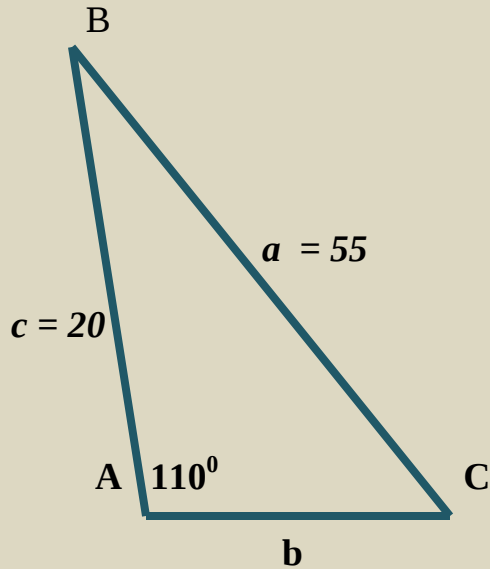
$$f/\sin F = d/\sin D \rightarrow 24/(1/5) = d/(2/5)$$

$$1/5d = 2/5(24)$$

$$d = 5/1 \cdot 2/5(24) \text{ (Use the rules of algebra to isolate the 'd' variable on one side)}$$

$$d = 48$$

Answer: 48



C. In the diagram, $a = 55$, $c = 20$, and $m\angle A = 110^\circ$. Find the measure of $\angle C$ to the nearest degree.

Use the Law of Sines:

$$a/\sin A = c/\sin C \rightarrow 55/\sin 110^\circ = 20/\sin C$$

$$55 \sin C = 20(\sin 110^\circ)$$

$$\sin C = 20(\sin 110^\circ)/55$$

$$\sin C = 20(0.940)/55$$

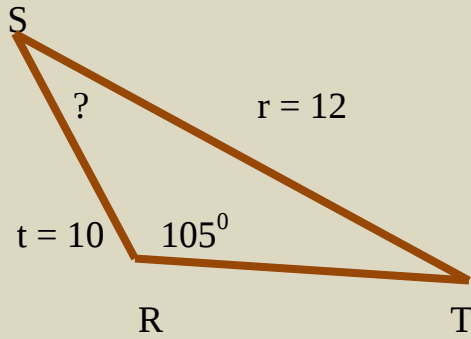
$$\sin C = .342$$

Unfortunately, this is NOT the answer!! We need the angle measure of C, not the $\sin C$. So there is one more step. This step illustrates how to find the value of the angle measure when you only have the sin.

NEW STEP: Using $C = \sin^{-1}(.342)$, we have $C = 19.999 = 20^\circ$

(Since triangle ABC already has an obtuse angle of 110 degrees, we can eliminate the notion that sin is also positioned in Quadrant II, which would give us a second obtuse angle.)

Answer: 20°



D. In $\triangle RST$, $m\angle R = 105^\circ$, $r = 12$, and $t = 10$. Find the $m\angle S$, to nearest degree.

$$\sin R / r = \sin T / t$$

To find the value of $\sin R$, enter 105, then click on 'sin'

The value is .97 (rounded)

Now enter the values into the formula

$$.97/12 = \sin T/10$$

Solve the ratio

On the calculator, click $.97/12 = .08$

Then multiply by $.08 \times 10 = .8$

Click on 'inv' then click 'sin'

The value of angle T is 53.6

Note: You might have a slightly larger number in the decimal places, depending on how many places you rounded. If you did not round but went straight through on the calculator, the value is 53.6

So ~

$$\text{Angle } T = 53.6$$

$$\text{Angle } s = 180 - \text{Angle } R + \text{Angle } T$$

$$\text{Angle } s = 21.02$$



Explanation: To find the value of angle S:

Step 1: You must first have the values of angles S and T

Step 2: Use the law of sines to find the measure of angle T

Step 3: Substitute the given values into the formula

Step 4: Use algebra to solve for $\sin T$

Step 5: Use the inverse function to find the value of angle T

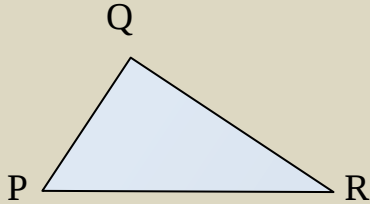
Step 6: Then subtract the measures of Angle r + Angle T from 180 degrees. This gives the value of angle S.

Answer = 21.02

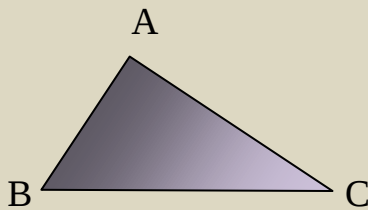


Practice Problems

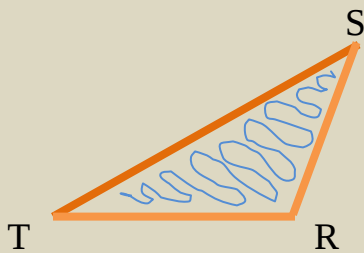
1. In $\triangle PQR$, $\sin P = 0.3$, $\sin R = 0.4$ and $r = 10$. Find the length of p .



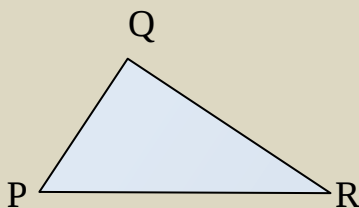
2. In $\triangle ABC$, $m\angle B = 22^\circ$, $m\angle C = 52^\circ$ and $a = 30$. Find the length of b to the nearest tenth.



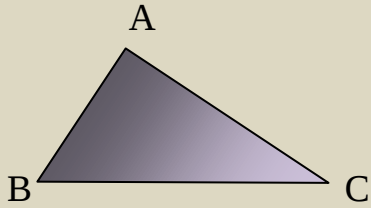
3. In $\triangle RST$, $m\angle R = 104^\circ$, $r = 20$, and $t = 10$. Find the $m\angle S$, to nearest degree.



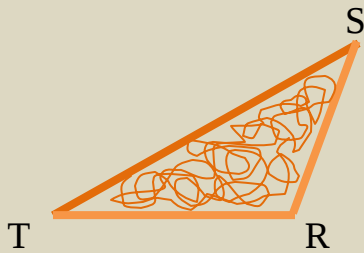
4. In $\triangle PQR$, $\sin P = 0.2$, $\sin R = 0.2$ and $r = 12$. Find the length of p .



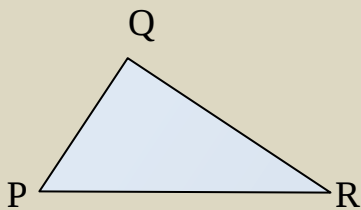
5. In $\triangle ABC$, $m\angle B = 12^\circ$, $m\angle C = 32^\circ$ and $a = 32$. Find the length of b to the nearest tenth.



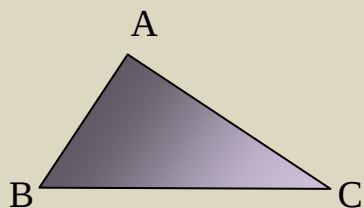
6. In $\triangle RST$, $m\angle R = 100^\circ$, $r = 20$, and $t = 12$. Find the $m\angle S$, to nearest degree.



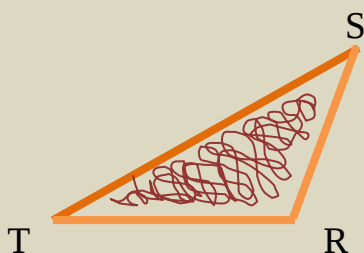
7. In $\triangle PQR$, $\sin P = 0.5$, $\sin R = 0.3$ and $r = 13$. Find the length of p .



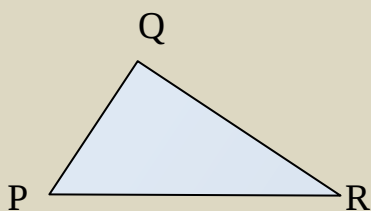
8. In $\triangle ABC$, $m\angle B = 14^\circ$, $m\angle C = 42^\circ$ and $a = 34$. Find the length of b to the nearest tenth.



9. In $\triangle RST$, $m\angle R = 98^\circ$, $r = 24$, and $t = 14$. Find the $m\angle S$, to nearest degree.



10. In $\triangle PQR$, $\sin P = 0.4$, $\sin R = 0.5$ and $r = 14$. Find the length of p .



Submit your answers in the text box.